ASSIGNMENT SET – I

Mathematics: Semester-I

M.Sc (CBCS)

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

PAPER - MTM-101

Paper: Real Analysis

1. Check whether the function
$f(x) = |x-1| + |x|$ on $[0,3]$ is a function of bounded variation or not. If so also find the variation function of f on $[0,3]$.
2. Show that collection M of all measurable set is a σ algebra.
3. The function f is defined on $[0,1]$ by
$f(x) = \begin{cases} n, if\ x \in Q^c \cap [0,1] \\ 0, if\ x \in Q \cap [0,1] \end{cases}$ where n denotes the number of zeros immediately after the decimal scale .show that f is measurable and find $\int f\, dx$.
4. Let f be an extended real valued function whose domain D is measurable . Then show that following are equivalent –
   (i) $\{x \in D: f(x) > \alpha\}$ is measurable .
   (ii) $\{x \in D: f(x) \geq \alpha\}$ is measurable .
   (iii) $\{x \in D: f(x) < \alpha\}$ is measurable .

5. Prove that outer measure of an interval is its length.

6. If $m^*(E) = 0$ then prove that E is a measurable set.

7. Let f be a continuous function defined on a closed set D. Then prove that f is measurable .

Is converse also true? Justify your answer.

8. Define outer measure of a set $A \subseteq \mathbb{R}$.

9. If f(x) is measurable defined on a measurable domain D and $f \sim g$ on D then prove that g(x) is also measurable .

10. Define characteristic function defined on a set $A \subseteq \mathbb{R}$.

11. Prove that characteristic function defined on set A is measurable if and only if A is measurable . 4

12. What is σ algebra .

13. Prove that outer measure of cantor set is zero.

14. check if E is measurable or not of the following sets

(i) $E = \left\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = \frac{1}{K^2} \; for\ all\ K \in \mathbb{N}\ and\ x \in \mathbb{Q}\ and\ y \in \mathbb{Q}\right\}$

(ii) $E = \mathbb{Q} \times \mathbb{Q} \times \ldots . \times \mathbb{Q}\ (100\ times)$

15. If is measurable then prove that $E + x$ is measurable and $m(E + x) = m(E)$ 2

16. Establish a necessary and sufficient condition for a function $f: [a, b] \to \mathbb{R}$ to be a function of bounded variation on $[a, b]$. 5

17. Show that the set of all functions of bounded variation on $[a, b]$ forms a vector space under usual addition and multiplication by scalars.

18. State and prove Monotone Convergence theorem.

19. Show that every bounded measurable function on $[a, b]$ is Lebesgue integrable on $[a, b]$. 5

20. Calculate the Lebesgue integral for the function

$$f(x) = \begin{cases} 1 & , when\ x\ is\ rational \\ 2 & , when\ x\ is\ irrational \end{cases}$$

21. If $f(x) = \begin{cases} \frac{1}{x} & , if\ 0 < x \le 1 \\ 9 & , if\ x = 0 \end{cases}$ then show that f is not Lebesgue integrable. 4

22. Show that outer measure of a singleton set is zero.

23. Show that every path connected metric space is connected. Give an example to show that the converse is not true .

24. Show that if a metric space X is compact then it is closed and bounded 5

25. Let $\{E_n\}$ be a sequence of measurable set such that

$$E_1 \subset E_2 \subset \quad \ldots . \subset \ E_n \subset \ \ldots ..$$

and if $E = \bigcup_{n=1}^{\infty} E_n$ then prove that $\mathrm{m(E)} = \lim_{n\to\infty} m(E_n)$

26. If a bounded function f is Lebesgue integrable on $[a, b]$ then $|f|$ is Lebesgue integrable over $[a, b]$. Moreover if f is Lebesgue integrable then

$$|\int_a^b f| \le \int_a^b |f|$$

27. Prove that the interval $(0, \infty)$ is measurable

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